Path Planning for CNC Machines Considering Centripetal Acceleration and Jerk

Abbas Shahzadeh
Tehran, Iran
shahzadeh@zapaleri.com

Abbas Khosravi, Saeid Nahavandi
Centre for Intelligent Systems Research
Deakin University
Geelong, 3217, VIC, Australia
{abbas.khosravi, saeid.nahavandi}@deakin.edu.au

***Abstract*— In planning an s-curve speed profile for a computer numerical control (CNC) machine, centripetal acceleration and its derivative have to be considered. In a CNC machine, these quantities dictate how much voltage and current should be applied to servo motor windings. In this paper, the necessity of considering centripetal jerk in speed profile generation especially in the look-ahead mode is explained. It is demonstrated that the magnitude of centripetal jerk is proportional to the curvature derivative of the path known as "sharpness". It is also explained that a proper limited jerk motion is only possible when a G2-continuous machining path is planned. Then using a simplified mathematical representation of clothoids, a novel method for approximating a given path with a sequence of clothoid segments is proposed. Using this method, a semi-parallel G2-continuous path with adjustable deviation from the original shape for a sample machining contour is generated. Maximum permissible feed rate for the generated path is also calculated.**

CNC; trajectory planning; s-curve; limited jerk; centripetal jerk look-ahead; clothoids; sharpness

I. INTRODUCTION

Many CNC controllers still use trapezoidal speed profile due to its simplicity and robustness. Figure 1a shows an example of this profile [1]. In this kind of profile, speed increases linearly from zero to a maximum speed at the beginning and decreases from the maximum speed to zero at the end of the profile. The main drawback of this method is that it requires a step change in acceleration. In a CNC machine, acceleration is determined by the electrical currents flowing in motor windings. So a step change in acceleration requires a step change in motor currents. However the windings usually have high inductance values which prevent fast changes of the currents flowing in them. So practically, accurate following of a trapezoidal speed profile is impossible.

On the other hand, derivatives of motor currents depend on the voltages applied to the windings. These voltages are controlled by fast electronic switches and can change in a fraction of a millisecond. As a result, regarding the physics of the system, the only parameter that can have an almost instantaneous change is the derivative of acceleration which is known as "jerk". Maximum applicable jerk is limited by a few factors including maximum motor voltage. Contrary to trapezoidal profiles, limited jerk speed profiles (also known as S-curve profiles) respect physical limitations of the system by bounding the maximum applied jerk. Figure 1b shows a sample s-curve profile [1].

Many papers have been published proposing different algorithms for generating limited jerk speed profiles [1]. In these papers, methods for limiting tangential jerk have been proposed. However, in most cases only bounding the tangential jerk might be insufficient.

When moving on curved paths, changes in centripetal acceleration can induce large jerk components on participating axes. The formula for centripetal acceleration $a=v^2/r$ is a well-known equation which states that this quantity depends on tangential velocity and the radius of curvature. In other words, centripetal acceleration and jerk depend not only on the speed profile, but also on the shape of the path.

In order to make the problem clearer, a vehicle moving in a highway can be considered as an example. For a vehicle, centripetal acceleration corresponds to the angle of front wheels and centripetal jerk is determined by the rate by which a driver turns the steering wheel. On a straight line, the radius of curvature is infinite. So the centripetal acceleration would be zero. Now suppose that the car goes suddenly from a straight line to a circular bend with radius of r_0. To stay on the bend, the driver has to turn the front wheels instantaneously in order to apply a centripetal acceleration equal to $a=v^2/r_0$. Obviously an instantaneous reorientation of the wheels is physically impossible. So the car has to stop, reorient its front wheels and start moving again [2]. This problem will happen for paths consisted of line segments and arcs because of curvature discontinuity. Planning G2-continuous paths, on which the radius of curvature never changes suddenly, can prevent this problem.

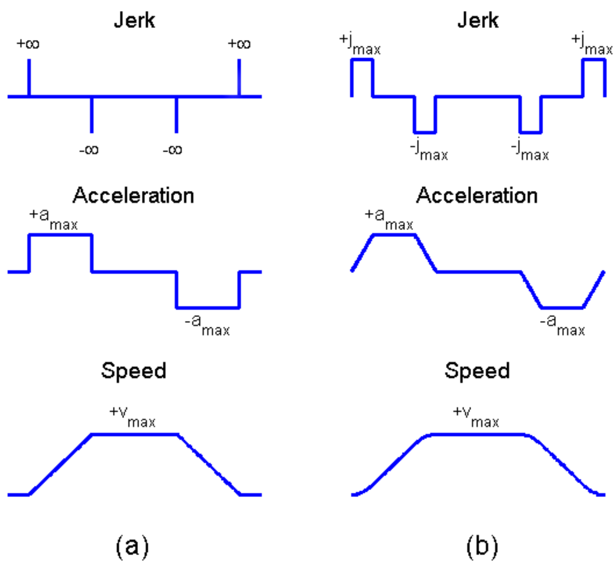

Figure 1. Trapezoidal speed profile (a) versus s-curve speed profile (b). Derivative of acceleration is limited in b.

There are two modes of operation in CNC machines known as "Look-ahead" and "Exact Stop". In the latter mode, no consideration of the machining path is required because the machine stops after travelling on each line or arc and starts moving again on the next one. But in the look-ahead mode, in order to have a true limited jerk motion, a G2-continuous path with limited sharpness (derivative of curvature) will be required. However the given machining path is not necessarily G2-continuous. So an approximation of the path with limited deviation from the main path has to be generated. A method for generating such approximation will be presented in the next sections.

The rest of this paper is organized as follows. Section II briefly reviews methods proposed in literature for G2-continuous path generation. Calculation of jerk is described in Section III. Section IV introduces a method for fitting a clothoid segment. How to generate a G2-continuous path using the proposed method is discussed in Section V. Finally, Section VI concludes the paper.

II. RELATED WORK

Methods for generating G2-continuous paths using clothoids are proposed in a number of papers. Mostly, these methods are intended for vehicles like wheeled robots [2], [3], [4], [5], autonomous guided vehicles [6], autonomous underwater vehicles [7], and airplanes [8]. CNC machines are different in respect of the feasible path. For a CNC machine, the preliminary path is already given and limiting the deviation from the initial path is the first priority. Furthermore maximum permissible errors are much smaller in CNC applications. Besides, vehicles normally move on straight lines and G2-continuous path planning is necessary for maneuvers like lane changing, overtaking and parking [9]. This is not the case for CNC machines.

In [10] a method for converting a piecewise linear curve to a G2-continuous path using Bezier curves is presented. The method can satisfy error limit constraints of CNC applications. However the method is rather complicated and divides the initial path into some groups and applies different methods to each group.

The method explained in [11] can be used in CNC machines as well. Using this method, an existing path can be smoothed while respecting maximum error constraint. But the resulting sharpness might be high since the curvature changes to zero in the middle of every line segment. This problem has been avoided in the method proposed in this paper.

III. REQUIRED JERK CALCULATION

A clothoid is a curve whose curvature is a linear function of the curve's length. These curves can be used to provide a smooth transition between curves with different curvatures. In this section, the jerk vector generated due to changes in centripetal acceleration, when moving on a clothoid will be discussed. Both circular arcs and line segments can be considered as special cases of clothoid segments. Consequently, jerk formula, in the case of a circular motion, is a special case of the extracted equation. Notation used to describe a clothoid is taken from [11].

In a clothoid, according to the definition, the curvature can be expressed as (1) where $\kappa_0=1/r_0$ is the initial curvature, c is the rate of change of curvature known as sharpness, and s is the arc length of the clothoid.

$$\kappa(s) = \kappa_0 + cs \tag{1}$$

It is assumed that the clothoid is a transition between a curve with curvature equal to $\kappa_0=1/r_0$ to another curve with curvature equal to $\kappa_1=1/r_1$ as depicted in figure 2. It's also assumed that the initial tangent angle is θ_0. Curvature by definition is the derivative of tangent angle $\theta(s)$ to the curve length s. Thus, the equation for $\theta(s)$ can be derived by integrating (1) as stated in (2).

$$\theta(s) = \theta_0 + \int_0^s \kappa(t)dt = \theta_0 + \kappa_0 s + \frac{1}{2}cs^2 \tag{2}$$

Having the tangent angle $\theta(s)$ from (2) and the magnitude of tangential velocity, vector of tangential velocity can be written as (3) in which, $\hat{\imath}$ and $\hat{\jmath}$ are the unit vectors codirectional with x and y axes, $|v|=ds/dt$ is the magnitude of tangential velocity and $\hat{T}$ is the unit tangent vector.

$$\begin{gathered}\overrightarrow{V(s)} = |v| \times \hat{T} \Rightarrow \\ \overrightarrow{V(s)} = |v| \times \left(cos(\theta(s))\hat{\imath} + sin(\theta(s))\hat{\jmath} \right) \Rightarrow \\ \overrightarrow{V(s)} = |v| \times \begin{pmatrix} \cos\left(\theta_0 + \kappa_0 s + \frac{1}{2}cs^2\right)\hat{\imath} + \\ \sin\left(\theta_0 + \kappa_0 s + \frac{1}{2}cs^2\right)\hat{\jmath} \end{pmatrix}\end{gathered} \tag{3}$$

By differentiating (3) with respect to time, acceleration vector can be derived as given in (4):

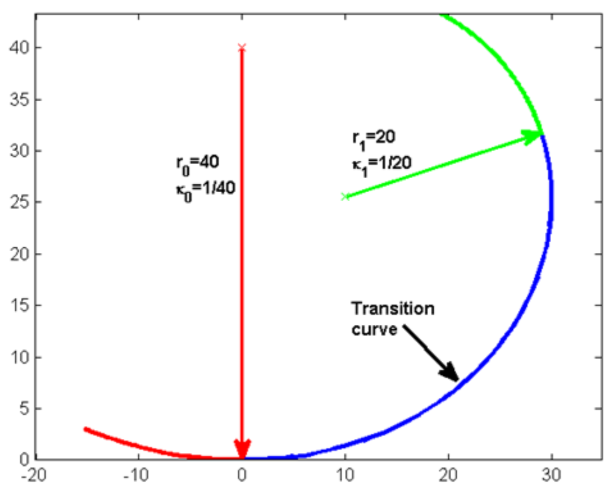

Figure 2. A clothoid (the curve in the middle) used as a transition curve connecting two circular arcs with different radii. Curvature of the clothoid changes linearly from $1/r_0$ to $1/r_1$.

$$\overrightarrow{a(s)} = \frac{d\vec{V}}{dt} = \frac{ds}{dt} \times \frac{d\vec{V}}{ds} \Rightarrow$$
$$\overrightarrow{a(s)} = |v| \times |v| \times (\kappa_0 + cs) \times \begin{bmatrix} -\sin\left(\theta_0 + \kappa_0 s + \frac{1}{2}cs^2\right)\hat{\imath} \\ +\cos\left(\theta_0 + \kappa_0 s + \frac{1}{2}cs^2\right)\hat{\jmath} \end{bmatrix} \quad (4)$$

With respect to (4) and considering the fact that the expression in the square brackets, is the tangential unit vector implicitly defined in (3) rotated by 90 degrees, the equation can be written in the more concise form of (5).

$$\overrightarrow{a(s)} = (\kappa_0 + cs)|v|^2 \times R(90°) \times \hat{T} \quad (5)$$

In the case of a circular motion, (5) will turn into $a = \kappa_0|v|^2 = |v|^2/r$ which is the well-known centripetal acceleration formula. Finally the equation for jerk is stated in (6).

$$\overrightarrow{jerk(s)} = \frac{d\vec{a}}{dt} = \frac{ds}{dt} \times \frac{d\vec{a}}{ds} \Rightarrow$$
$$\overrightarrow{jerk(s)} = c|v|^3 \times R(90°) \times \hat{T} + (\kappa_0 + cs)^2|v|^3 \times R(180°) \times \hat{T} \quad (6)$$

According to (6), the jerk vector has two components. The magnitude of the centripetal component is $c|v|^3$. This shows that the faster the curvature of an arc changes, the greater jerk on participating axes would be required. The tangential component is not of interest in this paper but has to be considered in speed profile generation.

Equations derived in this section show that in order to limit the required jerk, both curvature and its derivative should be bounded. This can be achieved by careful planning of machining path. Nevertheless, in addition to planning a proper tool path, in every section of the contour, feed rate should be limited according to the curvature and the rate of change of curvature to prevent violating maximum allowable acceleration and jerk. According to (5) and (6), feed limits imposed by curvature and sharpness are given in (7), (8) and (9) in which a_{max} and j_{max} are the maximum permissible acceleration and jerk respectively. κ_{max} is the maximum curvature (minimum radius) on the path and c_{max} is the maximum curvature derivative (sharpness) of the path. f_{max1} is the feed limit due to centripetal acceleration and f_{max2} is the limit due to centripetal jerk.

$$f_{max1} = \sqrt{a_{max}/\kappa_{max}} \quad (7)$$
$$f_{max2} = \sqrt[3]{j_{max}/c_{max}} \quad (8)$$
$$f_{max} = min(f_{max1}, f_{max2}) \quad (9)$$

IV. Clothoid Fitting

In this section a method for fitting a clothoid segment between a start point and a line will be presented. The clothoid will end to a point with ($x(s)$, $y(s)$) coordinates on the line and it should form a right angle with the line at the end point. A sample line and the perpendicular clothoid are displayed in figure 3. Generally the clothoid can have an initial curvature of κ_0 and an initial angle of θ_0 with the positive direction of x axis at the start point. The line is described by $y=mx+b$ equation. This method will be used in the next section for generating a G2-continuous path.

Coordinates of any point on a clothoid can be calculated by integrating the right side of equation (3) as stated in (10),(11) and (12):

$$\overrightarrow{P(s)} = x(s)\hat{\imath} + y(s)\hat{\jmath} \quad (10)$$
$$x(s) = x_0 + \int_0^s cos\left(\theta_0 + \kappa_0 t + \frac{1}{2}ct^2\right)dt \quad (11)$$
$$y(s) = y_0 + \int_0^s sin\left(\theta_0 + \kappa_0 t + \frac{1}{2}ct^2\right)dt \quad (12)$$

Using (11) and (12) coordinates of the end point of the clothoid segment can be calculated. In these equations, $\overrightarrow{P(s)}$ is the position vector of the end point (or any point corresponding to arc length equal to s), (x_0, y_0) are the coordinates of the start point of the clothoid segment, κ_0 is the curvature at the start point, θ_0 is the tangent angle at the start point, c is the sharpness of the clothoid and s is its length. All parameters except c and s are given. Two constraints have to be satisfied: end point should lie on the line and clothoid should be perpendicular to the line at the end point. First constraint implies that $x(s)$ and $y(s)$ should satisfy the line's equation i.e. $y=mx+b$. The first constraint is expressed in (13). For two shapes to form a right angle, product of their slopes should equal -1. This second constraint is formulated in (14) in which m is the line's slope.

$$y(s) = m.x(s) + b \quad (13)$$
$$\frac{dy(s)/ds}{dx(s)/ds}.m = -1 \quad (14)$$

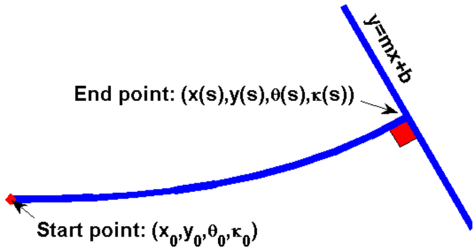

Figure 3. Fitting a clothoid between a start point and a line. The end point lies on the line and the two shapes form a right angle at the end point.

Replacing $x(s)$ and $y(s)$ from (11) and (12) in (13) and (14), yields equations (15) and (16):

$$y_0 + \int_0^s sin\left(\theta_0 + \kappa_0 t + \frac{1}{2}ct^2\right)dt = \\ m \times \left(x_0 + \int_0^s \cos\left(\theta_0 + \kappa_0 t + \frac{1}{2}ct^2\right)dt\right) + b \quad (15)$$

$$tan\left(\theta_0 + \kappa_0 s + \frac{1}{2}cs^2\right).m = -1 \quad (16)$$

In (15) and (16), c (sharpness of the clothoid) and s (arc length of the clothoid) are the only unknown variables. Equation (15) and equation (16) are derivable in respect to c and s. The two unknown variables can be calculated by Newton method in just a few iterations. After finding c and s, coordinates of the end point ($x(s)$, $y(s)$) can be calculated using (11) and (12). The curvature and the angle of the clothoid at the end point can be calculated using (1) and (2) respectively.

V. Generating a G2-Continuous Path

Using the method explained in the previous section, a G2-continuous path, semi-parallel to the given path can be generated. Using (5) and (6) the maximum feed rate on the approximating path can be calculated. The proposed algorithm will be described by using an example. Consider a machine with maximum feed rate of 3000 mm/min on X and Y axes. If motors can generate enough torque for the axes so that this feed rate can be reached in one second, maximum acceleration would be a_{max}=50 mm/s^2. Given that this acceleration can be gained in 0.1 s, maximum achievable jerk would be j_{max}=500 mm/s^3. Figure 4 shows the sample desired machining path. It shows a rounded corner which happens frequently in CNC milling. Obviously the path lacks curvature continuity and can't be followed by the machine without complete stops at A and B points marked in the picture.

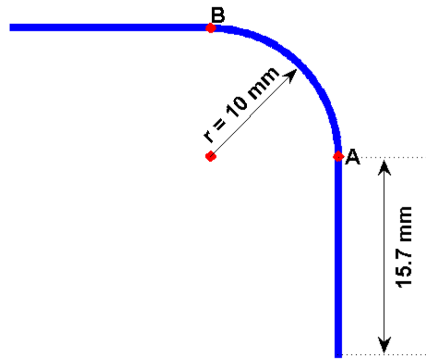

Figure 4. Sample machining contour. With a limited jerk speed profile, machine has to stop at A and B and start moving again.

Firstly the path should be divided into a number of segments. In figure 5, the shape is divided into 6 segments using 7 points marked in the picture. In figure 5, every three adjacent points form an angle. Bisectors of these angles are displayed using dotted lines. Fitted clothoids are perpendicular to these lines. The first clothoid has been fitted between the start point of the shape and the bisector of the first angle using the method described in the previous section. Next clothoid has been fitted between the end point of the first clothoid and the bisector of the second angle using the same method and so on. For every clothoid equations (15) and (16) has been solved and values for sharpness and arc length have been found.

The generated path has a maximum distance of 0.94 mm with the original path. Curvature changes have been depicted in figure 6. Maximum sharpness happens in the 3rd and 4th clothoids and is $c_{max}=0.0084$. According to (8), the feed rate limit due to sharpness on these clothoids will be $\sqrt[3]{j_{max}/c_{max}}$=2342 mm/min. As expressed in (7) maximum curvature – or minimum radius - imposes another limit on the feed rate equal to $\sqrt{a_{max}/\kappa_{max}}$=1271 mm/min which is the dominant limit in this case.

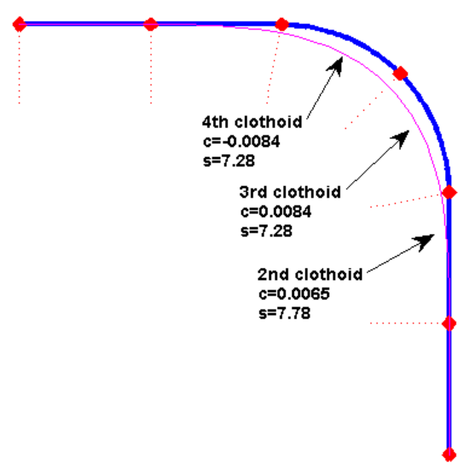

Figure 5. Generating approximating G2-continuous path. The clothoids make right angles with dotted lines. Sharpness and arc length values (c and s) for a few clothoids are reported in this picture.

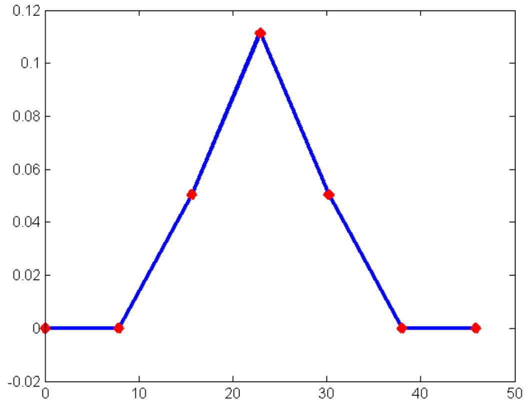

Figure 6. Curvature profile of the generated path.

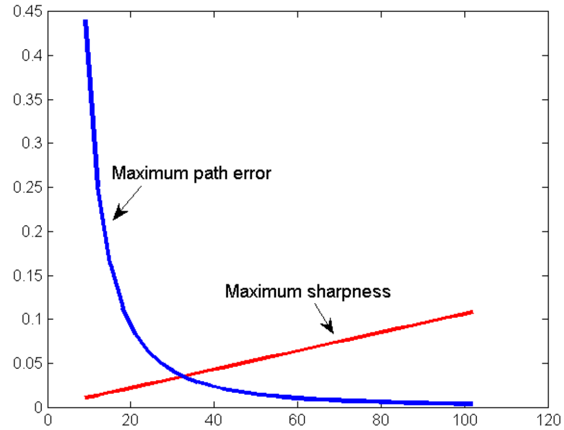

Figure 7. Maximum path error and maximum sharpness of the G2-continuous path generated by the proposed method versus number of used clothoid segments. By increasing the number of segments, sharpness increases linearly while error decreases quadratically.

It should be noted that usually machining is carried out in two phases: roughing cycle and finishing cycle. The introduced error in the example case is relatively large for finishing cycles. In the proposed method in order to decrease the error between the G2-continuous path and the given path, number of segments should increase. One undesirable by-product of shrinking segment sizes would be an increase in the sharpness which will result in reduced maximum permissible feed rate. The original shape has infinite values of sharpness at two points. So it is predictable that by approaching the shape, sharpness would increase.

Basically the proposed method generates a path semi-parallel to the original path. Two parallel shapes with one common point are actually coincident shapes and lie exactly on top of one another. Using the proposed method when the number of segments tends to infinity, the generated shape will be parallel to the original shape and consequently will lie exactly on top of it.

It was observed that by every doubling of the number of segments, error decreases almost fourfold while the sharpness increases approximately twofold. Maximum sharpness and maximum error versus segment number graphs for the example case are illustrated in figure 7. For the sample path, by increasing the number of segments to 96, maximum deviation decreased to 0.004 mm and maximum sharpness increased to 0.1020, imposing a feed rate limit of 476 mm/min. Curvature graph for this case is reported in figure 8.

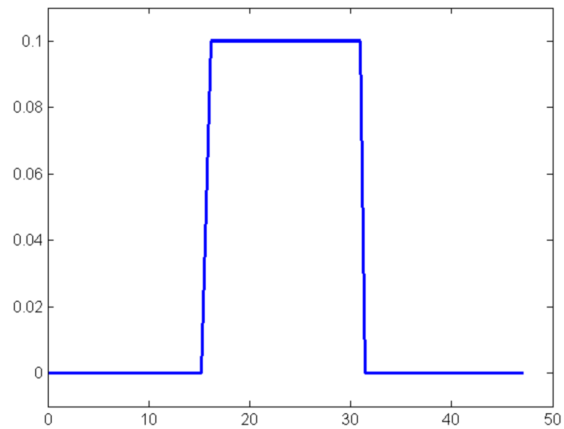

Figure 8. Curvature profile of the generated path when using 96 clothoid segmetns.

VI. CONCLUSION

The necessity of generating a continuous curvature path was described and equations explaining the relation between sharpness and centripetal jerk were derived. A method for fitting clothoids between a start point and a line was introduced. The method was used in the proposed algorithm for making a G2-continuous path approximating the original path. It was demonstrated through a few graphs that maximum deviation between the generated path and original path can be reduced by increasing the number of clothoid segments.

References

[1] K. D. Nguyen, I. Chen, and T. Ng, "Planning algorithms for s-curve trajectories," *2007 IEEE/ASME international conference on advanced intelligent mechatronics*, vol. 1, pp. 1–6, 2007.

[2] A. Scheuer and T. Fraichard, "Planning continuous-curvature paths for car-like robots," in *Proceedings of the 1996 IEEE/RSJ International Conference on Intelligent Robots and Systems '96, IROS 96*, 1996, vol. 3, pp. 1304–1311 vol.3.

[3] D. H. Shin and S. Singh, "Path Generation for Robot Vehicles Using Composite Clothoid Segments," Robotics Institute, Pittsburgh, PA, CMU-RI-TR-90-31, Dec. 1990.

[4] D. K. Wilde, "Computing clothoid segments for trajectory generation," in *IEEE/RSJ International Conference on Intelligent Robots and Systems, 2009. IROS 2009*, 2009, pp. 2440–2445.

[5] V. Girbes, L. Armesto, and J. Tornero, "On generating continuous-curvature paths for line following problem with curvature and sharpness constraints," in *2011 IEEE International Conference on Robotics and Automation (ICRA)*, 2011, pp. 6156–6161.

[6] A. Zelinsky and I. Dowson, "Continuous smooth path execution for an autonomous guided vehicle (AGV)," in *TENCON "92. "Technology Enabling Tomorrow : Computers, Communications and Automation towards the 21st Century." 1992 IEEE Region 10 International Conference.*, 1992, pp. 871–875 vol.2.

[7] Zoran, Vukic, Bernhard, Lampe, and Matko, Barisic, "Geometric Primitives-Based AUV Path Planning in Cluttered Waterspaces," 2010, pp. 22–27.

[8] G. Brambley and N. L. Fulton, "Specific Length Trajectories Optimised for Maximum Acceleration using Conic Parameterised Clothoids," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 47, no. 4, pp. 2627–2636, 2011.

[9] V. Girbes, L. Armesto, J. Tornero, and J. E. Solanes, "Continuous-curvature kinematic control for path following problems," in *2011 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2011, pp. 4335–4340.

[10] J. Selinger and L. Linsen, "Efficient Curvature-optimized G2-continuous Path Generation with Guaranteed Error Bound for 3-axis Machining," in *2011 15th International Conference on Information Visualisation (IV)*, 2011, pp. 519–527.

[11] M. Brezak and I. Petrovic, "Path Smoothing Using Clothoids for Differential Drive Mobile Robots," 2011, pp. 1133–1138.